Figure 1.11. Mathematical Teaching Practices Continuum

What do they look like in practice?

Teacher Centered	Making Progress	Student Centered
ESTABLISH MATHEMATICS GOALS TO FOCUS LEARNING.		
Students are specifically reminded of the mathematics they did previously and what they will do next. Teacher shows general rule or talks about the point of the lesson.	Mathematics goals are clearly or become clearly related to the mathematics students have been learning and the overall goals of the unit, grade level, or course and possibly to a familiar situation or future application. Lesson includes an opportunity for students to generalize, explain, and justify solutions. Mathematics goal drives the student-led summary.	Mathematics goals of the lesson are clear and connected to the learning progression or become clear and connected to the learning progression. Students can explain why they are doing a mathematical task, how it is related to other areas of math, and possibly some important uses. Lesson engages all students in generalizing, explaining, and justifying solutions. Mathematics goal drives the student-led summary and the connections. Students can explain why they are doing the task, how it is related to other areas of math, and possibly some important uses.
IMPLEMENT TASKS THAT PROMOTE REASONING AND PROBLEM SOLVING.		
Nature of mathematical **tasks is unidimensional** (e.g., narrows the focus of one's thinking), may be **focused on** a specific set of **procedures**, and/or does *not* support the need for a diverse set of group members' skills. Teacher continually lowers the cognitive demand of the task (e.g., **heavily scaffolding the task** such that the opportunity for problem solving is minimal).	Mathematical tasks are sufficiently complex and group-worthy, but the **nature of the way they are posed fails to draw the students into the mathematical work** to be done (e.g., no making conjectures, the teacher may either consciously or inadvertently focus more directly on the context in ways that detract from the main mathematical goal of the lesson, or the teacher presents the problem in ways that students approach the task as a "set of exercises" to be completed). **Task is posed in a way that invites speculation, but cognitive demand erodes throughout the lesson** (e.g., **heavily scaffolding the task,** reducing opportunity for problem solving; **providing *some* entrée into the solution path by asking leading questions;** or using explicit statements that lead the learners to use a certain approach [e.g., "you may want to rearrange your (x, y) table so that the rate of change is more obvious"]).	Nature of the mathematical **tasks is rich, appropriately challenging, complex,** and **lends to multiple entry points and solution pathways.** Tasks are posed in ways that **invite speculation.**

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<table>
<tr><th>Teacher Centered</th><th>Making Progress</th><th>Student Centered</th></tr>
<tr><td colspan="3">USE AND CONNECT MATHEMATICAL REPRESENTATIONS.</td></tr>
<tr><td>Teacher does not support connections to prior learning.
Teacher points out that students will need this mathematics for the homework or some future test or course.
The class does a warmup that practices something they will need for the day's lesson.
Representations are not connected to other representations or to mathematical models, written methods, or strategies.</td><td>Nature of how students share out their thinking seems more about turn-taking than a genuine consolidation of understanding (e.g., every group presents their answers while the class passively listens).
Teacher shows students how new learning fits in with what they have been learning.
Students are specifically reminded of the mathematics they did previously and what they will do next.
Representations may loosely be connected or discussed, or connections are unplanned and not explicitly connected to other representations, models, strategies, or written methods.</td><td>Students are making connections to previous knowledge, skills, and understandings.
Students have the opportunity to put new learning in context with other mathematics they have learned and discuss where the new learning might be useful.
Mathematical models and representations are explored and connected to the learning and to other representations to drive discussions around big mathematical ideas.</td></tr>
<tr><td colspan="3">FACILITATE MEANINGFUL MATHEMATICAL DISCOURSE.</td></tr>
<tr><td>Classroom arrangement does not support collaborative work (e.g., rows and columns with no opportunity for talking provided).
Teacher is fixed at the front of the classroom.
Lecture prevails rather than an interactive atmosphere.
Minimal (or no) opportunities for collaboration are provided.
Teacher provides minimal opportunities for students to share their own thinking or work with their peers.
The final mathematical authority clearly resides with the teacher.</td><td>Students may be sitting in groups, but there is minimal engagement between group members.
Teacher provides some opportunities for collaboration (e.g., pair work).
Exploration time is either too little or too much.
Teacher lacks the confidence to utilize instructional strategies that relinquish control of the classroom to students (still more focused on controlling student behavior versus gathering evidence of student learning).
Teacher appears to have anticipated common student misconceptions but may miss opportunities to surface them in ways that support a consolidated understanding of the concepts.
Students' arguments are focused on what they did, but not necessarily why they did what they did.</td><td>Teacher purposefully prompts students to talk about each other's explanations (purposeful critique).
Teacher strategically chooses what students share and there is purposeful sequencing to support the mathematical focus of the lesson.
Teacher seems to be more at ease with the management of a problem-centered, collaborative classroom (e.g., purposefully promotes group interaction).
Students question each other, and teacher encourages this behavior.
Students' arguments are focused on both how and why they did what they did.
Students are positive (supportive atmosphere in which students are helping students).
Teacher appears to have established a protocol/norm for the learning culture.
Teacher appears to be purposely monitoring and selecting students to share their presentations with the class.</td></tr>
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Teacher Centered	Making Progress	Student Centered
POSE PURPOSEFUL QUESTIONS.		
Teacher both asks **and answers his or her own questions.** Learning is passive (little to **no active student engagement**). Teachers asks **"fill-in-the-blank" questions** (and students appear to be guessing what to insert in the blank). Teacher **asks low-level** (e.g., recall) **questions.** **Teacher provides very little or no wait time.** **Teacher does all of the summarizing (goes into telling mode).** Teacher continually rephrases or revoices students' responses. Teachers continue to call on individual students until a student provides the response they were looking for.	Students are **not readily questioning or critiquing the reasoning of their peers.** **Teacher provides minimal opportunity for students to reflect** on their learning. Students are **narrowly focused on their own responses** rather than that of their peers. Teacher gathers data during the investigation but appears to **rely on volunteers during the discussion** (does not purposefully select and sequence shares). Students still seem **reliant on teacher's affirmation** of their approach.	**Students** are engaging in questioning the reasoning of their peers. **Students are thinking about efficiency** and are **naturally wondering about generalizations.** The **authority seems to reside in their reasoning and defense about the math** (rather than the teacher). **Students are thinking about efficiency** and are **naturally wondering about generalizations.** **Classroom culture seems to have fostered curiosity and sense making, which is reflected both in terms of the questions that students pose to one another and in the questions that students think about themselves ("I wonder if this always works?" "Why does this seem to be true?" "Can I find a counterexample?")** **Students build on one another's strategies/thinking and generate and defend arguments.**
BUILD PROCEDURAL FLUENCY FROM CONCEPTUAL UNDERSTANDING.		
Teacher asks for explanations of procedures, applications of rules, or use of definitions. The teacher guides students toward one strategy. The teacher leads the whole class in developing reasoning and justification for correct solutions. Students look only to the teacher or the text to validate mathematical correctness.	Teacher's questions probe reasoning about mathematical relationships, mathematical representations, and why solutions make sense. Questions build on what students start but tend to channel students' thinking toward a preferred strategy or method of solution. The teacher entertains alternative methods and solutions so long as students can justify their reasoning.	Teachers ask for justification of conjectures and encourage students to question and extend their own thinking to make new connections.

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Teacher Centered	Making Progress	Student Centered
SUPPORT PRODUCTIVE STRUGGLE IN LEARNING MATHEMATICS.		
Teacher **provides examples** of how to solve the task in advance of engaging students in solving the problem(s). (*Teaching as telling* …) Teacher gives **too many hints** and/or answers questions for the students. Teacher **does not provide adequate time for consolidation of learning.** Summary consists of a whole-group share-out with no time for processing and the **share-out is dominated by a small set of student volunteers.** **Opportunity for reflection on learning is nonexistent.** **Teacher, educational support professionals, or other students solve the problem for the student.**	Teacher provides **adequate individual think time,** which provides access and promotes productive contributions during group work, but **not all individuals in the class appear to be legitimately attending to it** during the individual think time (e.g., hands go up, spending the time writing details but not engaging in solving the problem, etc.). The **teacher prolongs the length of time utilized to launch/pose the problem,** and as a result, students appear to lose interest in attending to it when given the opportunity to engage in solving.	Students exhibit **perseverance** through engagement in classroom learning tasks. Students are comfortable making mistakes, critiquing and questioning each other, and **analyzing errors** (**safe environment** where students try out ideas). Students **understand their challenge** and appear to be **intent on attending to it.** **Teacher provides adequate individual think time,** which provides access and promotes productive contributions during group work. **Students leverage this time** to deepen mathematical understanding.
ELICIT AND USE EVIDENCE OF STUDENT THINKING.		
Teacher **relies on** a consistent **(small) group of volunteers.** Students ask to check answers or how to complete the task. The teacher demonstrates and justifies correct methods and solutions. Students look to teacher or text to validate correctness.	Teacher gathers data during the investigation but appears to **rely on volunteers** (does not purposefully select and sequence shares). Students are held **accountable for learning** (e.g., actively involved during share-out, taking notes, critiquing, asking questions), but the nature of the questions does not necessarily demand stronger argumentation. **Teacher provides minimal opportunity for students to reflect** on their learning. Students are focused on procedures. "I've done … what do I do? Next? How do you solve …?" The teacher leads the whole class in developing reasoning and justification for correct solutions. Students look to teacher or text to validate correctness.	Teacher **provides opportunities for additional thoughts/insights** and **questions.** **Mathematical proficiency** appears to be **evolving** over time. Teacher **anticipates, notes, and fully addresses** common student **misconceptions.** **Teacher purposely works at prompting and making student reasoning and thinking public (in the foreground).** **Students ask not just how to do something but also expect each other to explain why it works. "Why do you think that …?"** Students argue the validity of a mathematical statement or solution through reasoning and justifying. When they have solved a problem, students tend to believe they are correct and are ready to present their reasoning.

Source: Adapted from NCTM (2014, 2017); McGatha, Bay-Williams, Kobett, and Wray (2018); and Delaware Math Coalition (2013).

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